ΑΣΚΗΣΕΙΣ

1) Να υπολογιστούν τα ολοκληρώματα

i) $\int \left(\frac{x \cdot e^x}{(x+1)^2} \right) dx$ ii) $\int \left(\frac{3x^2-1}{2x\sqrt{x}} \, Arcεφ\, x \right) dx$

iii) $\int \left(e^{ημx} \frac{x \cdot συν^3 x - ημx}{συν^2 x} \right) dx$ iv) $\int \left((x^2-1) συν 3x \right) dx$

ΛΥΣΗ

i) $\int \frac{1}{(x+1)^2} dx = \int \left(-\frac{1}{x+1} \right)' dx = -\frac{1}{x+1}$ ⊛.

Άρα, ⊛ $\int x \cdot e^x \left(-\frac{1}{x+1} \right)' dx =$

$= x \cdot e^x \left(-\frac{1}{x+1} \right) + \int (e^x + xe^x) \frac{1}{x+1} dx =$

$= -\frac{x \cdot e^x}{x+1} + \int e^x \cancel{(x+1)} \cdot \frac{1}{\cancel{x+1}} dx =$

$= -\frac{x \cdot e^x}{x+1} + \int e^x dx = -\frac{x \cdot e^x}{x+1} + e^x + c.$

ii) $\int \frac{3x^2-1}{2x\sqrt{x}} dx = \int \left(\frac{3}{2} x^{1/2} - \frac{1}{2} x^{-3/2} \right) dx =$

$= \int \left(x^{3/2} + x^{-1/2} \right)' dx = x^{3/2} + \frac{1}{x^{1/2}} = \frac{x^2+1}{\sqrt{x}}$

Συνεπώς, $\int \left(\frac{x^2+1}{\sqrt{x}} \right)' Arcεφx \, dx =$

$$= \frac{x^2+1}{\sqrt{x}} \cdot \text{Arc}\,\varepsilon\varphi x - \int \frac{\cancel{x^2+1}}{\sqrt{x}} \cdot \frac{1}{\cancel{1+x^2}}\, dx =$$

$$= \frac{x^2+1}{\sqrt{x}}\, \text{Arc}\,\varepsilon\varphi x - \int \frac{1}{\sqrt{x}}\, dx =$$

$$= \frac{x^2+1}{\sqrt{x}}\, \text{Arc}\,\varepsilon\varphi x - \int \frac{2}{2\sqrt{x}}\, dx =$$

$$= \frac{x^2+1}{\sqrt{x}}\, \text{Arc}\,\varepsilon\varphi x - 2\sqrt{x} + C$$

iii) $$\int e^{\eta\mu x}\, \frac{x\cdot\sigma\upsilon\nu^3 x - \eta\mu x}{\sigma\upsilon\nu^2 x}\, dx =$$

$$= \int e^{\eta\mu x}\left(\frac{x\cdot\sigma\upsilon\nu^3 x}{\sigma\upsilon\nu^2 x} - \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}\right) dx =$$

$$= \int e^{\eta\mu x}\left(x\cdot\sigma\upsilon\nu x + \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}\right) dx =$$

$$= \int e^{\eta\mu x}\cdot x\,\sigma\upsilon\nu x\, dx + \int e^{\eta\mu x}\, \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}\, dx =$$

$$= \int x\left(e^{\eta\mu x}\right)' dx + \int e^{\eta\mu x}\cdot\left(\frac{1}{\sigma\upsilon\nu x}\right)' dx =$$

$$= x\cdot e^{\eta\mu x} - \int e^{\eta\mu x} dx + e^{\eta\mu x}\cdot\frac{1}{\sigma\upsilon\nu x} - \int e^{\eta\mu x}\,\cancel{(\eta\mu x)'}\cdot\frac{1}{\cancel{\sigma\upsilon\nu x}}\, dx =$$

$$= x\, e^{\eta\mu x} - \cancel{\int e^{\eta\mu x} dx} + e^{\eta\mu x}\,\frac{1}{\sigma\upsilon\nu x} - \cancel{\int e^{\eta\mu x} dx} =$$

$$= x\cdot e^{\eta\mu x} + e^{\eta\mu x}\,\frac{1}{\sigma\upsilon\nu x} + C$$

iv) $\int (x^2-1)\overbrace{\text{συν}\,3x}^{\varphi(x)}\,dx = \int (x^2-1)\left(\frac{\text{ημ}\,3x}{3}\right)' dx =$

$= (x^2-1)\cdot\frac{\text{ημ}\,3x}{3} - \int 2x\cdot\frac{\text{ημ}\,3x}{3}\,dx =$

$= (x^2-1)\cdot\frac{\text{ημ}\,3x}{3} - \frac{2}{3}\int x\,\text{ημ}\,3x\,dx =$

$= (x^2-1)\frac{\text{ημ}\,3x}{3} - \frac{2}{3}\int x\cdot\left(-\frac{\text{συν}\,3x}{3}\right)' dx =$

$= (x^2-1)\frac{\text{ημ}\,3x}{3} - \frac{2}{9}\left(-x\,\text{συν}\,3x + \int \frac{\text{συν}\,3x}{3}\,dx = \right.$

$= (x^2-1)\frac{\text{ημ}\,3x}{3} + \frac{2}{9}\,x\frac{\text{συν}\,3x}{3} + \int \frac{\text{συν}\,3x}{3}\,dx =$

$\Rightarrow = (x^2-1)\frac{\text{ημ}\,3x}{3} + \frac{2}{9}\,x\,\text{συν}\,3x - \frac{2}{27}\,\text{ημ}\,3x + c$